Change of Base

$\log_b c = \frac{\log c}{\log b} = \frac{\ln c}{\ln b}$

Use the change of base formula and your calculator to evaluate the logarithm

A) $\log_3 16$

$\frac{\log 16}{\log 3} = 2.534$

B) $\log_{1/2} 2$

$\frac{\log 2}{\log \frac{1}{2}} = -1$

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Express using only natural logarithms

A) $g(x) = \log_5 x$

$g(x) = \frac{\ln x}{\ln 5}$

B) $g(x) = \log_2(x+y)$

$g(x) = \frac{\log(x+y)}{\log 2}$

Find the exact solution algebraically, and check it by substituting into the original equation.

A) $\left(\frac{1}{4}\right)^{x}=\frac{1}{16}$

$x=2$

B) $20\left(\frac{1}{2}\right)^{x/3}=5$

$\frac{20}{20}$

$\left(\frac{1}{2}\right)^{x/3}=\frac{1}{4}$

$\left(\frac{1}{2}\right)^{x/3}=\left(\frac{1}{2}\right)^{2}$

$\frac{x}{3}=2$

$x=6$

$\frac{1}{4}=\left(\frac{1}{2}\right)^{2}$

C) $\frac{2(3)^{x/2}}{2}=\frac{6}{2}$

$3^{\frac{x}{2}}=3^{1}$

$\frac{x}{2}=1$

$x=2$

D) $\frac{2(3)^{-x/2}}{2}=\frac{54}{2}$

$3^{-x/2}=27$

$3^{-x/2}=3^{3}$

$-\frac{x}{2}=3$

$x=-6$

E) $\log x=5$

$10^{5}=x$

$x=100{,}000$

F) $\log_{2}(x-4)=3$

$2^{3}=x-4$

$8=x-4$

$x=12$

Solve each equation algebraically

A) $2.03^x = 5$

$\ln 2.03^x = \ln 5$

$x \ln 2.03 = \ln 5$

$x = \frac{\ln 5}{\ln 2.03}$

$= 2.27$

$\frac{\ln 5}{\ln 2.03} \neq \ln\left(\frac{5}{2.03}\right)$

B) $\frac{50(e)^{0.03x}}{50} = \frac{500}{50}$ $\ln e^1$

$e^{0.03x} = 10$

$\ln e^{.03x} = \ln 10$

$.03x \ln e = \ln 10$

$.03x = \ln 10$

$x = \frac{\ln 10}{.03}$

$x = 76.75$

C) $2\ln(x+3)+6=10$

$2\ln(x+3) = 4$

$\ln(x+3) = 2$

$e^2 = x+3$

$x = e^2 - 3$

$x = 4.39$

ln base e

$\log_e(x+3) = 2$

D) $2-\log(x+3)=10$

$-\log(x+3) = 8$

$\log(x+3) = -8$

$10^{-8} = x+3$

$x = -3 + 10^{-8}$